The Schur complement and its Statistical applications

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In multivariate statistics, we are very often confronted with problems of matrix calculation. Most of the associated calculations call upon a matrix division or Schur complements. We present various properties of matrix of the type

$$S = H - GE^{-1}F,$$

which we call the Schur complement of Ein

$$A = \begin{pmatrix} E & F \\ G & H \end{pmatrix}.$$

The matrix $E$is assumed to be nonsingular. When $E$is singular or rectangular we consider the generalized Schur compliment $S = H - GE^{g}F$where $E^{g}$is a generalized inverse of E. A comprehensive account of results pertaining to the determinant, the rank, the inverse and generalized inverses of partitioned matrices, and the inertia of a matrix is given both Schur compliments and for generalized Schur complements. We present the different properties of this division and make a non-exhaustive review of the associated results with a focus on statistics. We end this work with some applications in the field of road safety.

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